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On the supergravity evaluation of Wilson loop correlators in confining theories

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Abstract: We explicitly show the area law behavior of a circular Wilson loop in confining theories from supergravity. We calculate the correlator of two Wilson loops from supergravity in confining backgrounds. We find that it is dominated by an exchange of a "scalarball" that is lighter than the glueballs. We interpret these results in terms of the meson-meson potential in such theories.

Keywords: 1/N Expansion, Duality in Gauge Field Theories, p-branes.

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Contents

1. Introduction

In the AdS/CFT correspondence [1, 2, 3] correlation functions of gauge theory operators can be calculated by evaluating the supergravity action on $AdS_5 \times S^5$ with the gauge theory operators positioned on the boundary of AdS_5, acting as a source of supergravity fields. Every gauge theory operator has a corresponding supergravity mode.

The correspondence also holds when probes are introduced in AdS_5. A fundamental string with ends on the boundary is one such probe. It was shown in [4, 5] that the renormalized classical action of such a string translates into the expectation value of a Wilson loop in the dual gauge theory picture. Apart from the infinite strip loop which measures the potential between a quark-anti-quark pair, stringy computations were also made for circular loops [6, 7] and loops with cusps [8]. The incorporation of quadratic quantum corrections to the classical determination of the Wilson loops was addressed in [9, 11, 12].

The string worldsheet acts as a source of supergravity modes just like the gauge theory operators located on the boundary [2, 3]. In [7, 13, 14, 15] the supergravity action on $AdS_5 \times S^5$ was calculated in the presence of such a string worldsheet in order to get the expectation value of some operator $\mathcal{O}(x)$ in the presence of an external quark or a quark-anti-quark system. The action was calculated by considering the exchange of the corresponding supergravity mode from the worldsheet to the boundary point x.

The Wilson loop OPE and the correlation of two loops were calculated using the same method [7]. It was shown in [6, 17] that there is a solution of the string equation of motion that describes a surface that ends on two circles. This solution is stable only when the distance between the two loops is of the order of their radius. For longer distances the connecting world sheet degenerates into a thin tube [6].

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A natural question raised immediately after the evaluation of the Wilson loop of the $\mathcal{N}=4$ SYM theory, was how to extend these results to non-supersymmetric gauge dynamics. Witten [18] proposed to describe such theories using the near extremal solution, in the limit of zero radius of the compact Euclidean time direction combined with anti-periodic boundary conditions. This scenario renders the gauginos and adjoint scalars into very massive fields and hence resembles the pure YM theory. This approach was utilized to determine the behavior of the quark anti-quark potential for the $\mathcal{N}=4$ theory at finite temperature [24, 25] and the "pure YM" theory in three dimensions [23]. Later, a similar procedure was invoked to compute Wilson loops of four dimensional YM theory, 't Hooft loops [23, 6], the potential in MQCD and in Polyakov's type-0 model [19, 20, 21].

In [21] a unified scheme for such models was introduced and a theorem, that determines the leading and next to leading behavior, was proven and applied to several models. In particular a corollary of this theorem states the sufficient conditions for the potential to have a confining nature. Confining Wilson loops were also discussed in [10, 22].

The purpose of this paper is to analyze the two point function of Wilson loops associated with supergravity backgrounds that correspond to the large-N limit of non supersymmetric confining gauge theories. To achieve this goal we combine the approach of [7] with the recipe of [18]. The two point function of infinite strip Wilson loops translates into the interaction potential between mesons composed of heavy quarks in three dimensions. We find that the potential has the expected dependence on N and that it corresponds to an exchange of a "scalarball" which is lighter than the glueball. A similar approach can also be applied to determine the potential between baryons.

The paper is organized as follows. In section 2 we review the case of an infinite strip Wilson loops in the background of an AdS_5 "black hole", and calculate the circular Wilson loop area element. We explicitly show the area law in the circular case, and get the same string tension as in the strip case. In section 3 we calculate the correlation of two Wilson loops. We show that there are contributions from states lighter than the glueball states of YM_3 [29] indicating that the associated field theory dual is not pure YM theory in three dimensions. In section 4 we comment about extending the results to systems that include baryons. The results are summarized in section 5.

2. Circular Wilson loops

We start with a calculation of the area elements of circular Wilson loops in the background of N near extremal D3-branes, also known as the AdS black hole background [18].

$$ds^2 = \alpha'\left[\frac{u^2}{R^2}\left(1-\frac{u_T^4}{u^4}\right)dt^2 + \frac{u^2}{R^2}dx_{123}^2 + \frac{R^2}{u^2}\left(1-\frac{u_T^4}{u^4}\right)^{-1}du^2 + R^2 d\Omega_5^2\right]. \tag{2.1}$$

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The time coordinate is compactified on a circle of radius $1/T$. We will consider Wilson loops in the x_{12} plane, and constant θ on the S^5. In order to make the equations simpler we shall use $z = 1/u$, and rescale the coordinates as follows

$$\begin{aligned} \alpha' R^2 &= 1 \qquad \frac{x}{z_{\max} R^2} \to x\,, \\ \lambda &= \frac{z_{\max}^4}{z_T^4} \qquad \frac{z}{z_{\max}} \to z\,, \end{aligned} \tag{2.2}$$

where $z_{\max}$ is highest point in the worldsheet, and $z_T = (\pi R^2 T)^{-1}$. The metric is

$$ds^2 = \frac{1}{z^2}\Big[dt^2(1-\lambda z^4) + dx_{123}^2 + (1-\lambda z^4)^{-1} dz^2\Big] + d\Omega_5^2 \tag{2.3}$$

AdS_5 corresponds to $\lambda = 0$. The dilaton is independent of λ and z.

Infinite strip Wilson loops have been thoroughly investigated in this background [23, 25, 21, 26]. The worldsheet area element for a loop in the x-y plane is given by

$$d\mathcal{A} = \frac{1}{2\pi\alpha'} \frac{dy\, dz}{z^2\sqrt{1-z^4}\sqrt{1-\lambda z^4}}\,, \tag{2.4}$$

where dy is one of the spatial directions in the metric.

Circular Wilson loops can be calculated using similar methods. In an AdS_5 background circular Wilson loops have been investigated in [7, 6, 8]. By using the conformal symmetry on the boundary of AdS_5 one can transform a Wilson line into a circle, thus getting a simple analytic expression for the area of such a loop.

In non conformal cases, such as in the background of N near extremal D3 branes, one must use the standard approach [23, 25]. Our interest is circular loops in the x-y plane. We shall write $dxdy$ as $rdrd\theta$, and the integrand is the same as in the infinite strip case. The action gives the expression for the radius of the worldsheet as a function of z [18, 25]

$$r(z) = \int_z^1 \frac{z'^2 dz'}{\sqrt{1-z'^4}\sqrt{1-\lambda z'^4}}\,. \tag{2.5}$$

The difference in the circular case is that the boundary condition is $r(0) = a$, where a is the radius of the loop. The area element of the worldsheet is given by

$$d\mathcal{A} = \frac{1}{\alpha'} \frac{r(z) dz}{z^2\sqrt{1-z^4}\sqrt{1-\lambda z^4}}\,, \tag{2.6}$$

When integrating this area element the divergent part is proportional to the circumference of the loop and is subtracted to get a finite area. It can be seen that upon setting $\lambda = 0$, which is the extremal case, we get that the area is independent of the radius as was pointed out in [7]. In the limit $\lambda \to 1$ the dominant contribution to the area will be from $z = 1$, where we can write

$$d\mathcal{A} = \frac{1}{\alpha'} r(z) dr(z)\,. \tag{2.7}$$

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This gives the area law for circular loops $S = a^2/2$. Rescaling back the coordinates we get ($\lambda = 1$)

$$S = \frac{1}{2}\pi R^2 T^2 S_B\,, \tag{2.8}$$

where $S_B = \pi a^2$ is the area of the loop on the boundary. A similar expression was found in [23] for the connection between the energy and separation of a quark anti-quark system from the strip loop.

For a general metric the action will be of the general form

$$S = \frac{1}{\alpha'}\int r dr \sqrt{f^2(z) + g^2(z)(z')^2}\,, \tag{2.9}$$

with the boundary condition that the highest point in the worldsheet is $z = 1$. In this case the circular loop will have the following radius, and area

$$\begin{aligned} r(z) &= \int_z^1 dz' \frac{g(z')/f(z')}{\sqrt{f(z')^2/f^2(1) - 1}}\,, \\ d\mathcal{A} &= \frac{1}{\alpha'}\frac{r(z)g(z)f(z)}{f(1)\sqrt{f(z)^2/f^2(1) - 1}}\,. \end{aligned} \tag{2.10}$$

It can be seen that if $g(z)$ is singular at $z = 1$, then the dominant contribution to the radius will come from $z = 1$. Thus we can write S as

$$S = \frac{1}{\alpha'}\int f(1) r(z) dr(z) = \frac{f(1)a^2}{2\alpha'}\,, \tag{2.11}$$

in which case the area law behavior is evident. In case $g(z)$ is not singular in the $0 < z < 1$ interval, but $f(z)$ has a minimum at $z = 1$, the integrals in (2.10) are still dominated by the contribution at $z = 1$. The leading behavior will still be an area law for $f(1) > 0$.

3. Wilson loops correlators

It was shown by [13, 6] that when the separation of the two loops in AdS_5 is of the order of their size, there is a solution of the string equation of motion that describes a connected surface ending on the two loops (figure 1a). As the separation increases the tube shrinks, and becomes unstable. At large distances the correlation is due the exchange of supergravity modes in the bulk between the worldsheets of the loops.

The long distance correlation of two circular loops was calculated in [7] for the $\mathcal{N} = 4$ SYM theory. The long distance correlation is given by the exchange of light supergravity modes in the bulk of AdS_5 that couple to the worldsheet of the Wilson loops (figure 1b).

It seems plausible that the same picture would hold for other backgrounds that have field theory duals. In this note we are interested in the background created by N near extremal D3-branes.

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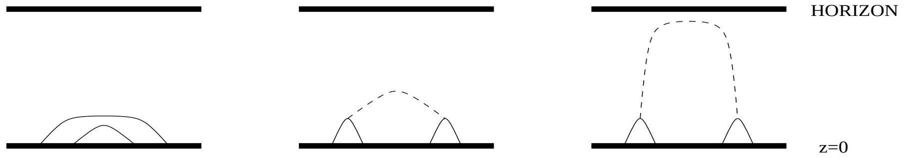

Figure 1: (a) The connected surface. (b) The exchange of supergravity modes between the worldsheets at distances $L \ll z_T$. (c) The exchange of supergravity modes at long distances.

Starting with ten dimensional supergravity compactified on $W \times S^5$ we may write the ten dimensional fields as

$$\Phi = \sum_{k,I} \phi_k Y_{k,I} \,, \tag{3.1}$$

where $\phi_{k,I}$ is a five dimensional field, and $Y_{k,I}$ are the spherical harmonics on S^5 with total angular momentum k. The spectrum of ten dimensional supergravity compactified on S^5 was investigated in [27, 28]. In what follows we will concentrate only on the dilaton, ϕ_k, and s_k, which is a linear combination of the trace of the metric on S^5 and the 4-form. These fields have the following five dimensional masses

$$\begin{aligned} \phi_k m_\phi^2 &= k(k+4)k \geq 0 \,, \\ s_k m_s^2 &= k(k-4)k \geq 2 \,. \end{aligned} \tag{3.2}$$

Note that the field s_k has for $k = 2, 3$ a negative m^2. However, these modes are not tachyonic, since they propagate on a space of negative curvature.

The general form of the correlation is obtained by integrating on the worldsheets the amplitude for the exchange of a supergravity mode in the bulk between points on the worldsheets, and then summing over all modes that can be exchanged [7].

$$\log \left[\frac{\langle W(0) W(L) \rangle}{\langle W(0) \rangle \langle W(L) \rangle} \right] = \sum_{i,k,I} Y_{k,I}^2 \int d\mathcal{A}_1 \int d\mathcal{A}_2 \, f_1^{i,k} \, f_2^{i,k} \, G^{i,k} \,, \tag{3.3}$$

where $f_1^{i,k}$, $f_2^{i,k}$ are the couplings of the field i to the worldsheet (in general they can be functions of z), k is the momentum on S^5, and $G^{i,k}$ is the propagator. The sum on I follows simply from the properties of spherical harmonic on S^5 [32].

We can read off the N, and $g_s N$ dependence of the correlation. The propagator derived from the supergravity action will be of order $\kappa^2 \sim \alpha'^4 g_s^2$. With our choice of units, and the factor of α'^{-2} coming from the worldsheet area elements, the amplitude will be of order $\alpha'^2 g_s^2 = g_s N / N^2$.

To get the leading contribution we shall consider only the lowest k's possible for each field. Note that the long distance correlation given by (3.3) does not depend on the relative orientation of the two loops, unlike the short distance correlation [13]. The worldsheet area elements in (3.3) are known for a general metric [21]. But for integrating (3.3) we need to know the propagator and the couplings in the AdS black hole background.

The coupling of ϕ_k and s_k in the case of a worldsheet in AdS_5 was found in [7]. The dilaton coupling follows simply from the relation between the Einstein and string frame metrics, $g_s = g_E e^{\phi/2}$. Therefore the one dilaton coupling is 1/2, and should be the same in the present case. The s_k coupling was found to be $f^{s,k} = -2kz^2/z_{\rm max}^2$ in AdS_5, but the coupling in a general metric is unknown to us. The couplings of supergravity modes in AdS_5 were also investigated in [16].

The effect of the horizon at $z = z_T$ comes in through the propagator, and the fact that the wave function of the exchanged supergravity modes should obey certain boundary conditions on the horizon, which makes the momentum in the z direction discrete, as was shown in [18]. At large distances, a particle exchanged between the two worldsheets will propagate parallel to the horizon. Therefore we assume that the full propagator will for large distances ($L \gg z_T$) be of the form

$$G(z_1, z_2, L) = F(z_1, z_2) G_3(L)\,, \tag{3.4}$$

where $G_3(L)$ is the flat three dimensional propagator, and all the z dependence is in the unknown function $F(z_1, z_2)$, where z_1 and z_2 are points on the worldsheets. This agrees with Witten's observation [18] that the exchange of supergravity modes in the five dimensional bulk corresponds to the exchange of field theory modes on the three dimensional boundary. The three dimensional mass is given by $M_i^2 \sim -p_i^2/z_T^2$, where p_i^2 are the discrete eigenvalues of the wave operator in the z direction.

In the high temperature limit the field theory scalars and fermions acquire masses. The dilaton which couples to $\operatorname{tr} F^2$ corresponds in field theory to the glueball states 0^{++}. Their masses were numerically calculated in [29, 30]. The field s_2 couples to the symmetric traceless tensor $\mathcal{C}_{IJ} \operatorname{tr} X^I X^J$, and corresponds to a "scalarball". When no external quarks, which are sources of both ϕ_0 and s_2, are present we can restrict ourselves correlation functions of the massless gauge fields, and only the glueball spectrum contributes. However, when introducing an external heavy quark we should consider the exchange of s_2 as well, since the worldsheet acts as its source. This is quite clear from the fact that the Wilson loop OPE [7] has both $\operatorname{tr} F^2$ and $\mathcal{C}_{IJ} \operatorname{tr} X^I X^J$ terms. This spoils the identification of the $T \to \infty$ limit with pure QCD_3. States like s_2, which have no counterparts in QCD, should be projected out by hand if one wishes to make such an identification. Note that even with no heavy quarks present the glueball masses [29] are of the same order as the scalar and fermion masses, and hence composites of the latter do not decouple.

Although the field s_2 has $m^2 = -4$ it can be shown numerically that all eigenvalues are negative, meaning that the three dimensional modes have positive M^2. The lowest mode is lighter than the lightest glueball. This is not surprising since it originated from a five dimensional mode with m^2 lower than the dilaton's. For example, the lowest eigenvalue found in [29] for the dilaton wave function was $p^2 = -11.59$. For the s_2 the lowest eigenvalue is $p^2 = -2.3$. We denote by M_s and M_ϕ the lightest three dimensional masses of s_2 and ϕ_0.

Circular loops. The correlation of two circular Wilson loops has two phases.

(i) For a small separation distance there is a connected solution.

(ii) For large distances the correlation is dominated by the exchange of s_2, as was the case in AdS_5 [7], but now using the 3 dimensional propagator and discrete masses we get for the lightest mass

$$\log\left[\frac{\langle W(0)W(L)\rangle}{\langle W(0)\rangle\langle W(L)\rangle}\right] \sim \frac{g_s N}{N^2}\frac{e^{-M_s L}}{L}\,. \tag{3.5}$$

The correlation of two circular loops in $\mathcal{N}=4$ SYM calculated using the AdS_5 bulk-to-bulk propagator gave [7]

$$\log\left[\frac{\langle W(0)W(L)\rangle}{\langle W(0)\rangle\langle W(L)\rangle}\right] \sim \frac{g_s N}{N^2}\frac{a^4}{L^4}\,, \tag{3.6}$$

where a is the radius of the loop.

Infinite strip loops in x-y plane. We can also calculate the correlation of two infinite strip loops in the x-y plane. We now have a worldsheet that is extended in one direction. Therefore the supergravity modes can be emitted and absorbed at any point along the y direction. This means that we should write $\sqrt{L^2+(y_1-y_2)^2}$ instead of L. According to (3.3) we should integrate over y_1, y_2.

$$\frac{1}{Y}\log\left[\frac{\langle W(0)W(L)\rangle}{\langle W(0)\rangle\langle W(L)\rangle}\right] \sim \frac{g_s N}{N^2}K_0(M_s L)\,, \tag{3.7}$$

where K_0 is a modified Bessel function, which has the following asymptotic behavior

$$K_0(M_s L) \approx \sqrt{\frac{\pi}{2M_s L}}e^{-M_s L}\,. \tag{3.8}$$

Infinite strip loops in x-t plane. The long distance correlation of two infinite strip loops in the x-t plane has the same L dependence as in the circular case. Although the worldsheet is extended in the t direction, it is not a dynamical direction in the $T\to\infty$ limit. In calculating the three dimensional spectrum we assumed zero momentum in the compact t direction.

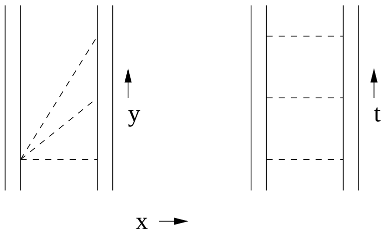

Figure 2: (a) Correlation of two strip loops in the x-y plane. (b) Correlation of two strip loops in the x-t plane.

The interest in the infinite strip Wilson loops comes from the fact that we can relate the expectation value of the correlation of two such loops to the potential between two heavy quark mesons. The potential in the $\mathcal{N}=4$ SYM case was calculated in [7], and was found to be of the form expected from QCD calculation [31].

$$V_{mm} \sim \frac{1}{Y} \log \left[\frac{\langle W(0)W(L) \rangle}{\langle W(0) \rangle \langle W(L) \rangle} \right]. \tag{3.9}$$

The infinite strip loop in the x-t plane corresponds to a meson in the $\mathcal{N}=4$ theory at finite temperature. The strip loop in the x-y plane corresponds in the $T \to \infty$ limit to a meson in a "QCD$_3$-like" theory. In order to make the identification with QCD$_3$ we must project out states like s_2 that have no counterparts in QCD$_3$.

For $L \gg z_T$ we get the following potentials

$$\begin{aligned} 4d\ T>0 \qquad & V_{mm} \sim \frac{g_s N}{N^2} \frac{e^{-M_s L}}{L}, \\ QCD_3 \qquad & V_{mm} \sim \frac{g_s N}{N^2} K_0(M_\phi L). \end{aligned} \tag{3.10}$$

These are the expected potentials in theories with a mass gap, and the specified dimensionality.

The generalization to other backgrounds [36] is more subtle. The background of other Dp-branes is not of the form $W \times S^5$. Therefore the exchanged particles have Kaluza-Klein masses that vary with their position (since the radius of the sphere is a function of z). One should also make an analysis of the spectrum of type-IIB supergravity on the desired background.

4. Baryon correlators

The correlation of baryons can be calculated using the same methods. The baryonic configuration [33] in the $AdS_5 \times S^5$ background is composed of a baryonic vertex made out of a D5 brane that wraps the S^5 and a set of N strings stretching between the vertex and the boundary of AdS_5. In [34] a simplified picture of this system was introduced where the action is just the sum of the action of the wrapped D5 and the N strings. It was shown there that the baryonic vertex will be situated at a certain $z_0 \neq 0$ and the total action was found to be proportional to N times the action of a corresponding Wilson loop, namely $\sim -N\sqrt{g_s N}/L$ where L is the radius of the baryon. Improved calculations based on the BPS nature of the configuration were performed in [37]. For the baryonic configuration in the confining non supersymmetric background these BPS methods are not applicable. A simplified analysis of the three dimensional baryons derived from the large-T limit of the AdS_5 black hole background resulted in energy of the form $\sim N(R^2T^2)L$ [34].

Two such baryonic configurations can exchange supergravity modes that can be emitted and absorbed by the worldsheet of the strings, and the wrapped D5-brane.

It is easy to realize that the L dependence of such an exchange is the same as in the Wilson loop correlations. However, the N dependence will now be different since each of the baryon worldsheets is N times that of a corresponding Wilson loop. The meson-baryon potential will be of order N^{-1}, and the baryon-baryon potential of order 1. Again, this is the expected behavior from theories with no dynamical quarks [35].

5. Discussion

The purpose of this project was to compute certain properties of confining large-N gauge theories in the framework of the dual supergravity picture. These properties include the expectation value of circular Wilson loops and the two point function of such loops and of infinite strip loops.

Whereas the renormalized circular Wilson loop in the $AdS_5 \times S^5$ background was shown to be independent of the area of the loop [7, 6, 8], the corresponding expectation value in the confining background admits an area law behavior similar to the result for the infinite strip in that background [23]. In [21] sufficient conditions for confinement where written down for a class of generalized metrics. In the present work we show that indeed loops that obey these conditions yield an area law behavior for the circular Wilson loop. It will be interesting to further generalize this result to smooth loops of arbitrary shape and to loops that include cusps [8].

A string dual to pure three dimensional YM theory was proposed in [18]. The idea is to put the $AdS_5 \times S^5$ background at finite temperature, impose anti-periodic boundary conditions and take the infinite temperature limit. The corresponding gauge picture is that of the $\mathcal{N} = 4$ SYM in the infinite temperature limit. Naively, it seems that the latter model corresponds to the pure YM theory in three dimensions because the gauge fields remain massless but the gauginos and the adjoint scalars acquire mass proportional to the temperature.

Our present work provides further evidence that in fact this is not the case. The gravity background corresponds to a confining theory but not that of the three dimensional pure YM theory. This follows from the calculation of the meson-meson interaction that is dominated by an exchange of the s_2 mode rather than the dilaton. The s_2 mode, which corresponds in the finite temperature $\mathcal{N} = 4$ description to a bound state of two scalars, and is obviously absent from the pure YM spectrum, is lighter than the dilaton that corresponds to the glueball 0^{++}. It will be interesting to work out the exact couplings of modes like s_2 to the string worldsheet in a general background. This will enable an explicit calculation of the correlator dependence on the size and shape of the loop. We argue that a similar structure of exchange interaction dominates the potential between two baryons.

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